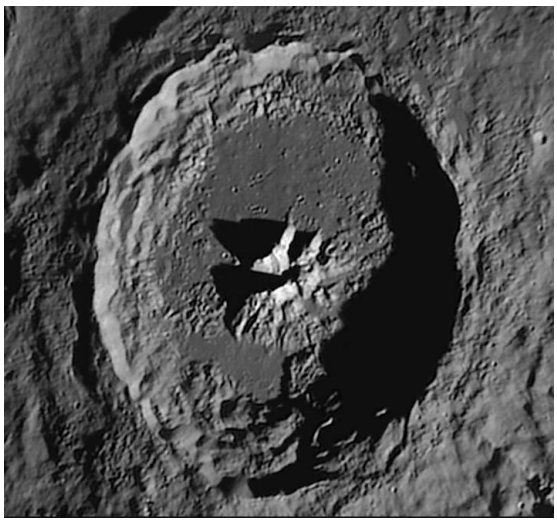
Calculating the Height of Lunar Features by Measuring Shadows

This is a simplified process and makes several assumptions (listed below) but it should give you something within the right ball park.

To begin with, you need a photo with clear shadows. In this example I've used a photo of the crater Theophilus taken by Alessandro Bianconi from Italy at 03:48 UT 18th September 2011, as shown below.

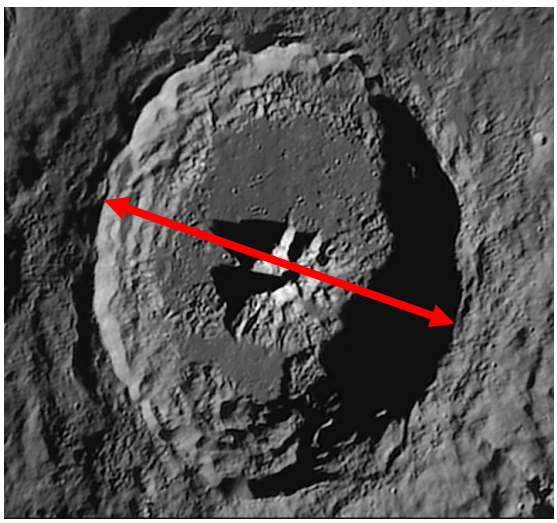


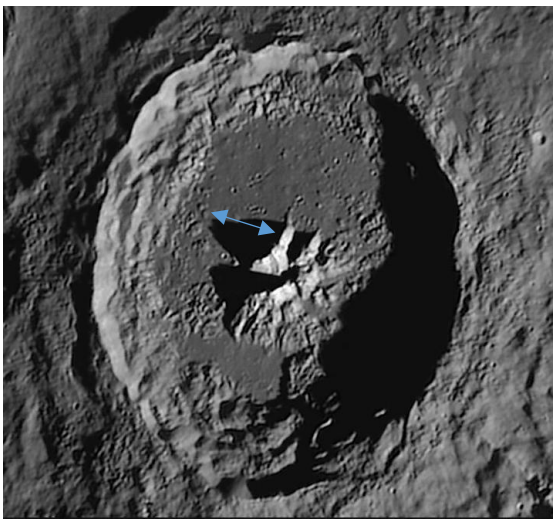
Before you start, you need to know how many metres per pixel the imaging set up has so you know the distance across the field of view of the photograph. If you're measuring a crater, then it's a bit easier because many crater diameters are published online.

Taking the Measurements

To keep the maths simple, the photo was zoomed so that the entire crater measured 10cm / 0.1m on my laptop screen; this makes one step of the maths a bit easier, but you can make the crater any size you like onscreen as long as you keep it the same for all measurements.

The shadows were measured with a ruler held up to my laptop screen (for a more precise measurement, you can open it in Photoshop and use the measure tool).





Calculations:
The crater diameter (red arrow) measured 0.1m on the laptop screen

The length of the shadow (blue arrow) measured 0.02m
The published diameter of Theophilus is 110,000 m

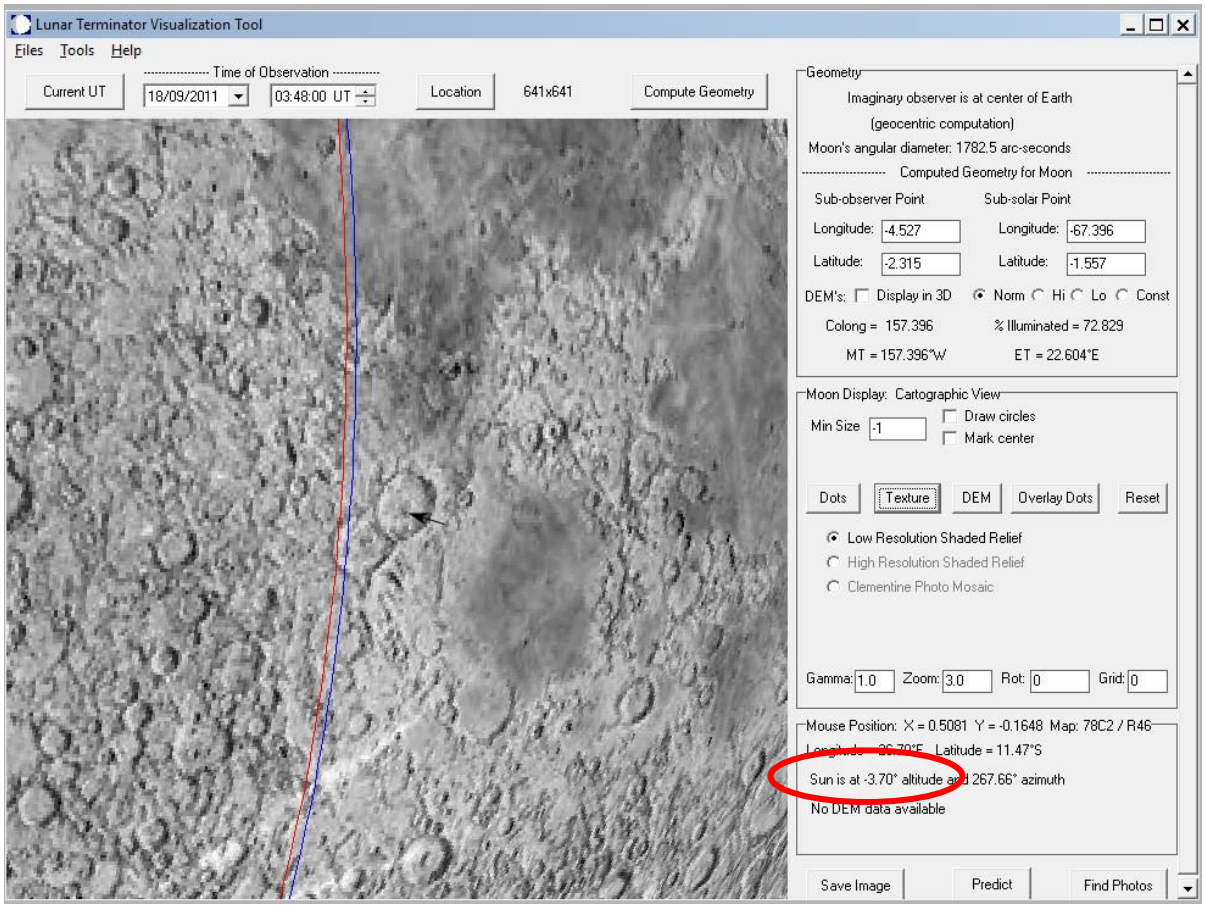
To calculate the real length of this shadow, divide the published crater diameter by the diameter on screen, then multiply by the shadow length:

110,000 / 0.1 x 0.02 = 22,000 m

The Lunar Terminator Visualisation Tool was then used to find the Sun angle at this feature at the time of the observation. You can download it from here:

https://github.com/fermigas/ltvt/wiki

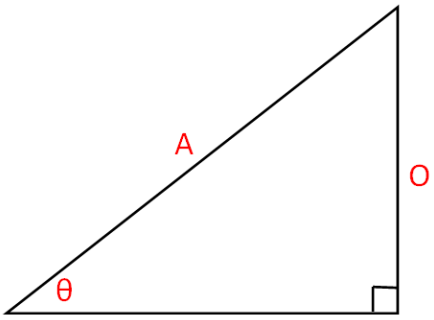
The date and time were entered at the top, then the mouse pointer was moved to point at the central peak on Theophilus. On the lower right side of the screen, the Sun angle where your mouse pointer is will be displayed (see screenshot below).



The Sun angle was found to be 3.7 degrees. Zooming in on the picture will give a more accurate Sun angle. Enter 2 or 3 into the “Zoom” box then click “texture” to zoom in.

There is a simple right angled triangle equation that can be used to calculate the height of features using their shadows:

$\tan\theta = O / A$



Where:
tanθ = tangent of the Sun angle (in radians)
O = height of the feature
A = length of the shadow

Once the Sun angle and shadow length are known, the equation can be rearranged and used to calculate the height of the feature:

$O = \tan\theta \times A$

To convert degrees into radians, divide pi by 180 then multiply by the angle in degrees:

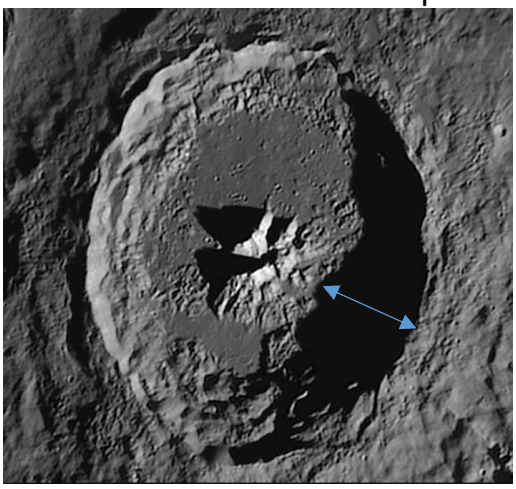
$\pi / 180 \times 3.7 = 0.6457718$ radians

To finish the calculation:

$O = \tan 0.6457718 \times 22{,}000$

O = 1,423 m
The published height is 1,400m.

The calculation was then repeated for part of the crater wall (blue arrow)



The measured shadow length was 0.03m

To calculate the actual length:

110,000 / 0.1 x 0.03 = 33,000 m

The Sun angle from the Lunar Terminator Visualisation Tool was 4.61 degrees

Converted to radians:

$\pi / 180 \times 4.61$ degrees = 0.08045968 radians

To finish the calculation:

O = tan 0.08045968 x 33,000

O = 2,661 m

The published height range is 3,200 – 4,200 m

All of these calculations were done using Excel as this reduces rounding errors. To save time, you can set up a spreadsheet template then just enter in the appropriate measurements. This will save time and allow you to perform a series of calculations in a much quicker timescale.

NB: The assumptions and simplifications made during this process

1. It can be difficult to know exactly where a feature or shadow starts and ends. Slump terraces and uneven crater walls make it challenging to measure the crater diameter or get accurate shadow measurements. Craterlets and other features on the floor of a crater may obscure the end of a shadow. It's also important that the shadow is measured in a straight line away from where the Sun is pointing

2. The published depth of a crater is usually an average figure, and often it is published as a range rather than an absolute number. Crater walls are not uniform, so if the published figure is an average and you're measuring the longest or shortest shadows, then you will invariably get a different answer to the published one

3. The floor of a crater may not be level. If it's sloping up or downhill then the shadow measurement will not be a 90 degree angle

4. The lunar surface is spherical, but the above calculations do not correct for the curvature. This problem will be more of an issue when measuring large craters and long shadows

5. A single measurement taken under one Sun angle will not be as accurate as taking multiple measurements and averaging them. Make sure you check the Sun angle for each measurement because it will be different as you move along the edge of the crater. You can then also repeat the entire process with different Sun angles during a crater sunrise or sunset, then take the average of all of the data. You should find it reduces the errors

I hope this was helpful. Good luck!